University of Colorado at Denver

Overlapping Domain Decomposition Method for Nonselfadjoint Elliptic Problems Discretized by Crouzeix-Raviart Elements

Jin-sheng Gu and Xian-cheng Hu

February 1995

UCD/CCM Report No. 42

CENTER FOR COMPUTATIONAL MATHEMATICS REPORTS

Overlapping Domain Decomposition Method for Nonselfadjoint Elliptic Problems Discretized by Crouzeix–Raviart Elements‡

Jinsheng Gu and Xiancheng Hu
(Dept. of Applied Math., Tsinghua University, Beijing 100084, P.R. China)

Abstract

Based on the discrete maximum principle established in this paper, we show the geometric convergence of the parallel Schwarz alternating method for numerically solving second order nonselfadjoint elliptic problems discretized by Crouzeix–Raviart nonconforming finite elements. The relation between its convergence factor and the subdomain division is also given.

§1 Introduction

With the development of parallel computers, the idea of domain decomposition[1] has been paid more and more attention in the area of large scale scientific computing. Being one of the most active branches of current computational mathematics, domain decomposition methods consist of parallel computing, preconditioning technique, multigrid, multilevel and fast algorithm[2]. Overlapping domain decomposition method, also known as the Schwarz alternating method, is the most original and fundamental domain decomposition. Based on the maximum principle and the theory of projection operator, P.L. Lions [3,4] studied the overlapping domain decomposition method for the second order elliptic continuous problems. When it was applied to the nonselfadjoint elliptic problems discretized by the conforming finite elements, its convergence was investigated respectively in [5,6] on the basis of GMRES (generalized minimum residual) and multigrid with the conclusion that it is efficient if the subdomain diameters are small enough. Later, D.L. Chu [6] considered its applications to solving the nonselfadjoint elliptic problems discretized by the piecewise linear continuous finite elements and proved its convergence with the discrete maximum principle, not requiring that the subdomain diameters should be small enough.

‡This work was supported by the National Natural Science Foundation of China under Grant No.19171050 and the Tsinghua University Fund for Science.

This paper presents the overlapping domain decomposition method for the nonselfadjoint elliptic problems discretized by the Crouzeix–Raviart elements. It is well–known that the Crouzeix–Raviart element[8] is the simplest element, which is often adopted in scientific and engineering computing. Many practical problems, such as the convection–diffusion problem, are the nonselfadjoint elliptic problems. Their nonselfadjoint property results in some difficulties in solving them numerically[9]. For our purpose, we first establish the discrete maximum principle, and then show the convergence of the parallel Schwarz alternating method for the nonselfadjoint elliptic problems discretized by the Crouzeix–Raviart elements. Finally, we point out that the more overlappingly the domain is decomposed, the more quickly the method converges.

§2 Discrete maximum principle

Consider the following nonselfadjoint elliptic problem

$$\begin{cases} -\Delta u + \beta(x)\cdot\nabla u + r(x)u = f, & \text{in } \Omega \\ u = 0, & \text{on } \partial\Omega \end{cases} \tag{2.1}$$

where $\Omega \subset \Re^2$ is a bounded polygonal domain, $\beta(x), r(x)$ are piecewise smooth and bounded in Ω, $r(x) \geq r_0 > 0, f \in H^{-1}(\Omega)$. The variational form of (2.1) is

$$u \in H_0^1(\Omega):\ a(u,v) = (f,v),\ \forall\ v \in H_0^1(\Omega) \tag{2.2}$$

where

$$a(u,v) = \int_\Omega \Big[\nabla u\cdot\nabla v + v\beta(x)\cdot\nabla u + r(x)uv\Big], \quad (f,v) = \int_\Omega fv.$$

By [10], we know that (2.2) has a unique solution.

Remark 2.1. For more general second order nonselfadjoint elliptic problem with the second order term $\sum\limits_{i,j=1}^{2} a_{ij}(x)\dfrac{\partial^2 u}{\partial x_i \partial x_j}$, there always exists an inverse transformation $\begin{cases} \xi_1 = \xi_1(x_1,x_2) \\ \xi_2 = \xi_2(x_1,x_2) \end{cases}$, such that $\sum\limits_{i,j=1}^{2} a_{ij}(x)\dfrac{\partial^2 u}{\partial x_i \partial x_j}$ can be transformed into $\dfrac{\partial^2 u}{\partial \xi_1^2} + \dfrac{\partial^2 u}{\partial \xi_2^2}$. Hence it is sufficient to study (2.1).

Suppose that $\Omega_h = \{e\}$ is a quasi–uniform mesh of Ω, such that for any triangular element $e \in \Omega_h$, each interior angle of e is not greater than $\frac{\pi}{2} - \alpha_0$, where α_0

is some positive constant. Let V_h be the Crouzeix–Raviart nonconforming finite element space[8] associated with Ω_h:

$$V_h = \Big\{v:\ v \text{ is continuous at each edge midpoint of } e,\ v|_e \text{ is linear }, \forall\ e \in \Omega_h\Big\}.$$

The Crouzeix–Raviart element discrete problem of (2.2) is

$$u_h \in V_h^0:\ A(u_h, v) = (f, v),\ \forall\ v \in V_h^0 \tag{2.3}$$

where
$$V_h^0 = \Big\{v \in V_h:\ v(x) = 0,\ \ \forall \text{ edge midpoint } x \in \partial\Omega\Big\},$$

$$A(w, v) = \sum_{e\in\Omega_h} \int_e \Big[\nabla w \cdot \nabla v + v\beta(x) \cdot \nabla w + r(x)wv\Big].$$

Let e be a triangular element as shown in Fig 2.1. $a_i, b_i, \mathrm{n_i}$ $(i = 1, 2, 3)$ denote the vertex, the edge midpoint and the unit outward normal vector of the opposite edge respectively. Let $|e|$ be the area of e and $\zeta_1 = |a_2a_3|, \zeta_2 = |a_3a_1|, \zeta_3 = |a_1a_2|$ be the lengths of its edges.

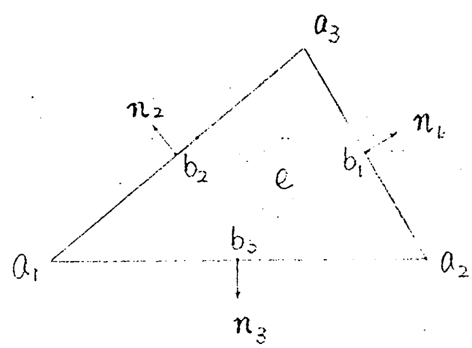

Fig 2.1

λ_i $(i = 1, 2, 3)$ is the area coordinate corresponding to the vertex a_i which satisfies that

$$\nabla\lambda_i = -\frac{\zeta_i}{2|e|}\mathrm{n_i} \tag{2.4}$$

The basis of Crouzeix–Raviart element associated with b_i is $\psi_i = 1-2\lambda_i$. Obviously, we have

Lemma 2.1. *Let α_0 be some positive constant. If each interior angle of $e \in \Omega_h$ is not greater than $\frac{\pi}{2} - \alpha_0$, then there exists a positive constant η, such that*

$$\int_e \nabla\psi_i \cdot \nabla\psi_j = \frac{\zeta_i\zeta_j}{|e|}\mathrm{n_i} \cdot \mathrm{n_j} \le -\eta, \quad 1 \le i \ne j \le 3. \tag{2.5}$$

Suppose that the edge midpoints of the elements $e \in \Omega_h$ are numbered in the following order: first those in Ω, denoted by $x_1, x_2, \cdots, x_m$, then those on $\partial\Omega$, denoted by $x_{m+1}, x_{m+2}, \cdots, x_{m+l}$. Let ϕ_j $(1 \le j \le m+l)$ be the basis with respect to x_j. It is easy to see that $V_h = Span\{\phi_j\}_1^{m+l}$, $V_h^0 = Span\{\phi_j\}_1^m$, $\sum_{i=1}^{m+l} \phi_i = 1$. Simple calculations imply that

$$A(\phi_i, \phi_i) = O(1) + O(h) + O(h^2),$$
$$\sum_{j=1}^{m+l} A(\phi_j, \phi_i) = \sum_{e \in \Omega_h} \int_e r(x)\phi_i = O(h^2) > 0, \quad 1 \le i \le m.$$

If x_i, x_j are not both on the boundary of any element $e \in \Omega_h$, then $A(\phi_j, \phi_i) = 0$; if x_i, x_j are different points on the boundary of some element, then it follows from Lemma 2.1 that

$$A(\phi_j, \phi_i) \le -\eta + O(h) + O(h^2).$$

Based on the above analysis, we come to

Lemma 2.2. *Suppose that each interior angle of $e \in \Omega_h$ is not greater than $\frac{\pi}{2} - \alpha_0$, where α_0 is some positive constant. Denote*

$$A = (a_{ij}) \in \Re^{m \times m}, \quad a_{ij} = A(\phi_j, \phi_i), \qquad 1 \le i, j \le m,$$
$$B = (b_{ik}) \in \Re^{m \times l}, \quad b_{ik} = A(\phi_{k+m}, \phi_i), \quad 1 \le i \le m, \ 1 \le k \le l.$$

If h is small enough, then

$$a_{ii} = O(1) > 0, \quad 1 \le i \le m, \tag{2.6}$$

$$a_{ij} \le 0, \quad 1 \le i \ne j \le m, \tag{2.7}$$

$$\sum_{j=1}^{m} a_{ij} + \sum_{j=1}^{l} b_{ij} = \int_\Omega r(x)\phi_i = O(h^2) > 0, \quad 1 \le i \le m, \tag{2.8}$$

$$b_{ij} \le 0, \quad 1 \le i \le m, \ 1 \le j \le l. \tag{2.9}$$

In what follows, $\max_\vartheta v$ means maximizing $v(x)$ over all the edge midpoints $x \in \vartheta$ for a measurable set ϑ.

Lemma 2.3. *Suppose that $\Gamma \subset \partial\Omega, \gamma \subset \Omega, \delta = dist(\gamma, \Gamma) > 0$. See Fig 2.2. There exists at least one edge midpoint on Γ and γ respectively. Let $w \in V_h$ which satisfies*

$$\begin{cases} A(w,v) = 0, & \forall\, v \in V_h^0 \\ w(x) = 0, & \forall \text{ edge midpoint } x \in \partial\Omega\backslash\Gamma \\ w(x) \leq 1, & \forall \text{ edge midpoint } x \in \Gamma \end{cases} \tag{2.10}$$

If each interior angle of $e \in \Omega_h$ is not greater than $\frac{\pi}{2} - \alpha_0$ (α_0 is some positive constant), then there exists a constant $\mu = O(1) > 0$, independent of h, such that for sufficient small h, we have

$$\max_{\gamma} w \leq \sigma = \exp(-\mu h\delta) < 1.$$

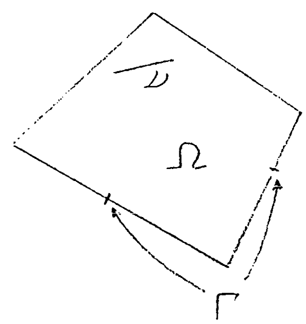

Fig 2.2

Proof. Let $x_0 \in \gamma$ be an edge midpoint. Construct $u \in V_h$, such that

$$u(x_i) = \exp[\tilde{\mu}(|x_i - x_0| - \rho)], \quad 1 \leq i \leq m + l. \tag{2.11}$$

Here $\rho = dist(x_0, \Gamma)$ and $\tilde{\mu}$ is some positive constant to be determined later. Denote

$$\begin{array}{ll} u_i = u(x_i), \ \ i = 1, 2, \cdots, m, & U = (u_1, u_2, \cdots, u_m)^T, \\ v_j = u(x_{j+m}), \ \ j = 1, 2, \cdots, l, & V = (v_1, v_2, \cdots, v_l)^T. \end{array}$$

If x_k, x_j are both on the boundary of some element, then $|x_k - x_0| \leq |x_j - x_0| + h$; otherwise, $a_{kj} = 0$ for $1 \leq k \neq j \leq m$ or $b_{kI} = 0$ for $1 \leq k \leq m, m+1 \leq j = I + m \leq m + l$. Lemma 2.2 gives

$$\begin{aligned} \sum_{j=1}^{m} a_{ij}u_j + \sum_{j=1}^{l} b_{ij}v_j \ &\geq u_i\Big[a_{ii} + \Big(\sum_{\substack{j=1 \\ j\neq i}}^{m} a_{ij} + \sum_{j=1}^{l} b_{ij}\Big)\exp(\tilde{\mu}h)\Big] \\ &= a_{ii}u_i\Big[1 - \Big(1 - \frac{1}{a_{ii}}\int_{\Omega} r(x)\phi_i\Big)\exp(\tilde{\mu}h)\Big], \ i = 1, 2, \cdots, m. \end{aligned}$$

In view of (2.6) and (2.8), we know that there exists a positive constant $\mu = O(1)$ such that for sufficient small h, $\tilde{\mu} = \mu h$ satisfies

$$1 - \left(1 - \frac{1}{a_{ii}} \int_\Omega r(x)\phi_i\right) \exp(\tilde{\mu} h) \geq 0.$$

Hence,

$$\sum_{j=1}^{m} a_{ij}u_j + \sum_{j=1}^{l} b_{ij}v_j \geq 0, \quad i = 1, 2, \cdots, m.$$

$$AU + BV \geq 0. \tag{2.12}$$

Let $w \in V_h$ be the solution of (2.10). Denote $W = (w(x_1), w(x_2), \cdots, w(x_m))^T$, $Z = (w(x_{m+1}), w(x_{m+2}), \cdots, w(x_{m+l}))^T$. It follows from (2.10) and (2.11) that

$$AW + BZ = 0, \tag{2.13}$$

$$0 \leq Z \leq V. \tag{2.14}$$

Lemma 2.2 indicates that $-B, A, A^{-1}$ are a nonnegative matrix, a M– matrix and a nonnegative matrix for sufficient small h respectively. Furthermore, using (2.12), (2.13) and (2.14), we have $W \leq U$. Therefore,

$$w(x_0) \leq u(x_0) = \exp(-\mu h \rho) = \exp(-\mu h dist(x_0, \Gamma)).$$

By the arbitrariness of $x_0 \in \gamma$, we end the proof of Lemma 2.3. □

Theorem 2.4. (Discrete maximum principle) *Suppose that* $\Gamma \subset \partial\Omega, \gamma \subset \Omega, \delta = dist(\gamma, \Gamma) > 0$. *See Fig 2.2. There exists at least one edge midpoint on* Γ *and* γ *respectively. Each interior angle of* $e \in \Omega_h$ *is not greater than* $\frac{\pi}{2} - \alpha_0$ *(*α_0 *is some positive constant). Suppose that* $\tilde{w}$ *is a function defined on the edge midpoints on* Γ. *If* $w \in V_h$ *satisfies*

$$\begin{cases} A(w, v) = 0, & \forall\, v \in V_h^0 \\ w(x) = 0, & \forall \text{ edge midpoint } x \in \partial\Omega \backslash \Gamma \\ w(x) = \tilde{w}(x), & \forall \text{ edge midpoint } x \in \Gamma \end{cases}$$

then there exists a positive constant $\mu = O(1)$, *independent of* h, *such that for sufficient small* h,

$$\max_\gamma |w| \leq \sigma \max_\Gamma |\tilde{w}| = \exp(-\mu h \delta) \max_\Gamma |\tilde{w}|.$$

Theorem 2.4 is the corollary of Lemma 2.3.

§3 Domain decomposition method

Decompose Ω into $\{\Omega_i\}_1^N$, the set of the open polygonal subdomains Ω_i, such that

A1. $\Omega = \bigcup_{i=1}^{N} \Omega_i$, $\partial\Omega_i \backslash \partial\Omega \subset \bigcup_{\substack{j=1 \\ j\neq i}}^{N} \Omega_j$, $i = 1, 2, \cdots, N$;

A2. If $i \neq j, j \neq k, k \neq i$, then $\partial\Omega_i \bigcap \partial\Omega_j \bigcap \partial\Omega_k = \emptyset$ and $\partial\Omega_i$ intersects $\partial\Omega_j$ at most two points;

A3. $\partial\Omega_i$ $(i = 1, 2, \cdots, N)$ is on the gridlines of Ω_h;

A4. For any edge midpoint $x \in \Omega_h$, x is the inner point of at most $N - 1$ subdomains.

For $i = 1, 2, \cdots, N$, the subspaces of V_h associated with Ω_i are denoted

$$V_h^i = \left\{v \in V_h : \ v(x) = 0, \ \forall \text{ edge midpoint } x \in \Omega \backslash \overline{\Omega}_i\right\},$$

$$V_h^{i,0} = \left\{v \in V_h : \ v(x) = 0, \ \forall \text{ edge midpoint } x \in \Omega \backslash \Omega_i\right\}.$$

The overlapping domain decomposition method for solving (2.3) is described as follows.

DDM3.1. (Parallel Schwarz alternating method):

Step 1. Choose arbitrarily $u^0 \in V_h^0$. Set $n := 0$;

Step 2. For $i = 1, 2, \cdots, N$, solve in parallel

$$\begin{cases} A(u_i^{n+1}, v) = (f, v), & \forall\, v \in V_h^{i,0} \\ u_i^{n+1} - u^n \in V_h^{i,0} \end{cases}$$

Step 3. Calculate $u^{n+1} = \frac{1}{N}\sum_{i=1}^{N} u_i^{n+1}$, set $n := n + 1$ and return to step 2 until some reasonable stopping criterion is satisfied.

§4 Convergence analysis

Let u_h be the solution of (2.3). u^n, u_i^n $(i = 1, 2, \cdots, N)$ are the iterative values of DDM3.1 at the nth iteration. Then $\varepsilon^n = u^n - u_h$, $\varepsilon_i^n = u_i^n - u_h$ $(i = 1, 2, \cdots, N)$ satisfy

$$\varepsilon^n = \frac{1}{N}\sum_{i=1}^{N} \varepsilon_i^n \in V_h^0, \tag{4.1}$$

$$\begin{cases} A(\varepsilon_i^n, v) = 0, & \forall\, v \in V_h^{i,0} \\ \varepsilon_i^n - \varepsilon^{n-1} \in V_h^{i,0} \end{cases} \tag{4.2}$$

Applying (4.1), (4.2) and Theorem 2.4 leads to

$$\max_{\Omega_i} |\varepsilon_i^{n+1}| \leq \max_{\partial\Omega_i} |\varepsilon_i^{n+1}| = \max_{\partial\Omega_i} |\varepsilon^n|,$$

$$\max_{\Omega} |\varepsilon_i^{n+1}| \leq \max\Big\{\max_{\Omega\backslash\Omega_i} |\varepsilon_i^{n+1}|, \max_{\Omega_i} |\varepsilon_i^{n+1}|\Big\} \leq \max_{\Omega} |\varepsilon^n|,$$

$$\max_{\Omega} |\varepsilon^{n+1}| \leq \max_{\Omega} |\varepsilon^n|. \tag{4.3}$$

For any subdomain Ω_i, it follows from A1 and A2 that $\partial\Omega_i\backslash\partial\Omega$ can be decomposed into s ($s \leq N-1$) segments: $\Gamma_i^1, \Gamma_i^2, \cdots, \Gamma_i^s$, such that $\partial\Omega_i\backslash\partial\Omega = \bigcup_{j=1}^{s} \Gamma_i^j$ and there exists $l_j \in \{1, 2, \cdots, i-1, i+1, \cdots, N\}$, such that $\Gamma_i^j \subset \Omega_{l_j}$, $\Gamma_i^j \bigcap (\partial\Omega_{l_j}\backslash\partial\Omega) = \emptyset$.

Theorem 2.4 indicates that

$$\max_{\Gamma_i^j} |\varepsilon_{l_j}^{n+1}| \leq \kappa \max_{\partial\Omega_{l_j}} |\varepsilon^n|, \; j = 1, 2, \cdots, s, \tag{4.4}$$

where $\kappa \in [0, 1)$ is some constant. In fact, κ is dependent on i, j and written as

$$\kappa_{ij} = \exp(-\mu h dist(\Gamma_i^j, \partial\Omega_{l_j}\backslash\partial\Omega)).$$

But for notational convenience, we omit the subscript of κ_{ij} here and later. It should be pointed out that although the constant κ might vary at different places, κ is in the interval [0,1).

By (4.1) and (4.4), we get

$$\begin{aligned} N \max_{\Gamma_i^j} |\varepsilon^{n+1}| &\leq \max_{\Gamma_i^j} |\varepsilon_1^{n+1}| + \cdots + \max_{\Gamma_i^j} |\varepsilon_N^{n+1}| \\ &\leq (N-1) \max_{\Omega} |\varepsilon^n| + \kappa \max_{\partial\Omega_{l_j}} |\varepsilon^n| \\ &\leq (N - 1 + \kappa) \max_{\Omega} |\varepsilon^n|. \end{aligned}$$

Hence,

$$\max_{\Gamma_i^j} |\varepsilon^{n+1}| \leq \frac{N-1+\kappa}{N} \max_{\Omega} |\varepsilon^n| \leq \kappa \max_{\Omega} |\varepsilon^n|,$$

$$\max_{\partial\Omega_i} |\varepsilon^{n+1}| \leq \max_{1\leq j\leq s} (\max_{\Gamma_i^j} |\varepsilon^{n+1}|) \leq \kappa \max_{\Omega} |\varepsilon^n|. \tag{4.5}$$

Furthermore, (4.3) yields

$$\max_{\bigcup_{i=1}^{N}\partial\Omega_i} |\varepsilon^{n+1}| \leq \kappa \max_{\Omega} |\varepsilon^n| \leq \kappa \max_{\Omega} |\varepsilon^{n-1}|. \tag{4.6}$$

For any edge midpoint $x \in \Omega \backslash \bigcup_{i=1}^{N} \partial\Omega_i$, there exist $I_1, I_2, \cdots, I_p \in \{1, 2, \cdots, N\}$, such that

$$x \in \Omega_{I_j},\ j = 1, 2, \cdots, p; \quad x \notin \Omega_i,\ i \in \{1, 2, \cdots, N\} \backslash \{I_1, I_2, \cdots, I_p\}.$$

It follows from (4.1), (4.3) and (4.5) that

$$\begin{aligned} N|\varepsilon^{n+1}(x)| &\leq (N-p)|\varepsilon^n(x)| + \max_{\partial\Omega_{I_1}} |\varepsilon^n| + \cdots + \max_{\partial\Omega_{I_p}} |\varepsilon^n| \\ &\leq (N-p)|\varepsilon^n(x)| + p\kappa \max_{\Omega} |\varepsilon^{n-1}|. \end{aligned}$$

A4 implies that there exist positive integers s_1, s_2 $(s_1 + s_2 = N)$, such that

$$\max_{\Omega \backslash \bigcup_{i=1}^{N}\partial\Omega_i} |\varepsilon^{n+1}| \leq \frac{s_1}{N} \max_{\Omega} |\varepsilon^n| + \frac{s_2}{N} \kappa \max_{\Omega} |\varepsilon^{n-1}|.$$

Now applying (4.3) gives

$$\max_{\Omega \backslash \bigcup_{i=1}^{N}\partial\Omega_i} |\varepsilon^{n+1}| \leq \kappa \max_{\Omega} |\varepsilon^{n-1}|.$$

Therefore, it follows from (4.6) that

$$\max_{\Omega} |\varepsilon^{n+1}| \leq \max\Big(\max_{\Omega \backslash \bigcup_{i=1}^{N}\partial\Omega_i} |\varepsilon^{n+1}|, \max_{\bigcup_{i=1}^{N}\partial\Omega_i} |\varepsilon^{n+1}| \Big) \leq \kappa \max_{\Omega} |\varepsilon^{n-1}|.$$

Based on the above analysis, we have

Theorem 4.1. *Suppose that each interior angle of $e \in \Omega_h$ is not greater than $\frac{\pi}{2} - \alpha_0$, where α_0 is some positive constant. Let u_h be the solution of (2.3) and u^n be the iterative value of DDM3.1 at the nth iteration. If h is small enough, then there exists a positive constant $\kappa \in [0, 1)$, such that*

$$\max_{\Omega} |u^{2n} - u_h| \leq \kappa^n \max_{\Omega} |u^0 - u_h|,$$

$$\max_{\Omega} |u^{2n+1} - u_h| \le \kappa^n \max_{\Omega} |u^1 - u_h|.$$

Furthermore, the higher the overlapping degree, the less the κ.

Now we investigate DDM3.1 from algebraic viewpoint. Denote $\hat{\Omega} = \{1, 2, \cdots, m\}$, $\hat{\Omega}_k = \{i \in \hat{\Omega} :$ edge midpoint $x_i \in \Omega_k\}$, $A_k = (A(\phi_j, \phi_i))_{i,j \in \hat{\Omega}_k}$. Let m_k be the number of the elements in $\hat{\Omega}_k$. Define the restriction operator $C_k : V_h^0 \to V_h^{k,0}|_{\Omega_k}$ as

$$\forall\, v \in V_h^0,\ C_k v \in V_h^{k,0}|_{\Omega_k},\ \big(C_k v\big)(x_j) = v(x_j),\ \forall\, j \in \hat{\Omega}_k.$$

C_k is in fact a matrix of order $m_k \times m$. It is easy to obtain the iterative matrix of DDM3.1[7]:

$$G = I - \frac{1}{N} \sum_{k=1}^{N} C_k^T A_k^{-1} C_k A. \tag{4.7}$$

Theorem 4.1 indicates that if each interior angle of $e \in \Omega_h$ is not greater than $\frac{\pi}{2} - \alpha_0$ (α_0 is some positive constant) and h is small enough, DDM3.1 converges geometrically with the spetral radius $\rho(G) < 1$. The higher the overlapping degree, the less the κ and $\rho(G)$. In this case, DDM3.1 converges more quickly, but the scale of each subproblem becomes larger correspondingly. It remains a problem how to decompose the domain Ω to balance the convergence rate and the computational work at each iteration.

It is known that A is a large scale sparse asymmetrical matrix (there are at most five non–zero elements at each row or column of A). Virtually, DDM3.1 is equivalent to making the simple iteration method converge through the construction of the iterative matrix as shown in (4.7) by selecting the main subblocks A_i ($i = 1, 2, \cdots, N$) which may have common elements among them. Therefore, DDM3.1 is efficient for solving large scale nonselfadjoint elliptic problems.

References

[1] H.A. Schwarz, Über einige Abbildungsaufgaben, Ges. Math. Abh., 11(1869), 65–83.

[2] T. Lü, et al., *Domain Decomposition Methods —New Numerical Techniques for Solving PDE,* (in Chinese), Science Press, Beijing, 1992.

[3] P.L. lions, On the Schwarz alternating method.I, in *Proceedings of First International Symposium on Domain Decomposition Methods for Partial Differential Equations*, R.Glowinski, et al.(eds), SIAM Pb., Philadelphia, 1988.

[4] P.L. lions, On the Schwarz alternating method.II, in *Domain Decomposition Methods*, T.F. Chan, et al.(eds), SIAM Pb., Philadelphia, 1989.

[5] X.C. Chan, *Some Domain Decomposition Algorithms for Nonselfadjoint Elliptic PDE's*, Ph.D. thesis, Courant Institute, 1989.
[6] D.L. Chu, *Some Problems in Domain Decomposition Methods for Elliptic Problems*, Ph.D. thesis, (in Chinese), Tsinghua University, 1991.
[7] S. Zhang, et al., Domain decomposition with overlapping and PCG method, J. Comp. Math., 1993.
[8] M. Crouzeix, et al., Conforming and nonconforming finite element methods for solving the stationary Stokes equations, RAIRO Numer. Anal., 7–R3(1973), 33–76.
[9] C. Johnson, et al., Crosswind smear and pointwise errors of streamline diffusion finite element methods, Math. Comp., 49(1987): 25–38
[10] S. Agmon, *Lectures on Elliptic Boundary Value Problems*, D. Van Nostrand Company, Inc., Princeton, 1965.

CENTER FOR COMPUTATIONAL MATHEMATICS REPORTS

University of Colorado at Denver
P.O. Box 173364, Campus Box 170
Denver, CO 80217-3364

Phone: (303) 556-8442
Fax: (303) 556-8550

26. C. Liao, Z. Liu and C. Liu, "Implicit Multigrid Method for Modeling 3-D Laminar Diffusion Flames with NOx Prediction."
27. Z. Cai, R. Lazarov, T.A. Manteuffel and S.F. McCormick, "First-Order System Least-Squares for Second-Order Partial Differential Equations: Part I."
28. Z. Cai, T.A. Manteuffel and S.F. McCormick, "First-Order System Least-Squares for Second-Order Partial Differential Equations: Part II."
29. S.E. Payne, "The Fundamental Theorem of q-Clan Geometry."
30. S.E. Payne, "A Tensor Product Action on q-Clan Generalized Quadrangles with $q = 2^e$."
31. J. Mandel, "Iterative Methods for p-Version Finite Elements: Preconditioning Thin Solids."
32. D.C. Fisher, K. Fraughnaugh and L. Langley, "P_3-Connected Graphs of Minimum Size."
33. J. Mandel and R. Tezaur, "Convergence of a Substructuring Method with Lagrange Multipliers."
34. P. Vaněk, J. Mandel and M. Brezina, "Algebraic Multigrid on Unstructured Meshes."
35. P. Vaněk and J. Křížková, "Two-Level Method on Unstructured Meshes with Convergence Rate Independent of the Coarse-Space Size."
36. P. Vaněk, J. Mandel and M. Brezina, "Algebraic Multigrid by Smoothed Aggregation for Second and Fourth Order Elliptic Problems."
37. H. J. Greenberg, "Mathematical Programming Models for Environmental Quality Control."
38. W.L. Briggs and V.E. Henson, "A Table of Analytical Discrete Fourier Transforms."
39. J.S. Gu and X.C. Hu, "Some Estimates with Nonconforming Elements in the Analysis of Domain Decomposition Methods."
40. J.S. Gu and X.C. Hu, "On Domain Decomposition Methods in Two-subdomain Nonoverlap Cases."
41. J.S. Gu and X.C. Hu, "A Simple Domain Decomposition Algorithm with Nonconforming Elements."