A Prospective on Mathematics and Artificial Intelligence: *Problem Solving = Modeling + Theorem Proving*

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Abstract

This is a prospective on the research in the intersection of mathematics and artificial intelligence that I see as having been the most important over the past 10 years and that I think should be pursued vigorously during this decade. Part of this is drawn from my personal research agenda, part is from vast readings, and part is from my editorial position with *Annals of Mathematics and Artificial Intelligence.*

Mathematics and artificial intelligence (AI) have had a symbiotic relationship since Allen Turing dreamed of taking Hilbert's tenth problem into the realm of computation that would blur the distinction between human and machine reasoning. Every aspect of AI has mathematical roots, and there have been some bilateral developments. For example, efforts to improve computational logic led to new results in mathematical logic, itself. Intelligent tutors have improved, particularly with modern programming paradigms, and some contribute to mathematics education. I see this as a part of mathematics because I view mathematics as not just a collection of facts, but as a process — *mathematical reasoning*. Once this is accepted, mathematics and artificial intelligence interleave throughout each of their branches.

As indicated in the title, problem solving involves two activities: modeling, sometimes called *abstraction* of a problem, and theorem proving. Both involve logical and analogical reasoning. Research into these has spawned several subdisciplines that have very similar goals and strategies, but which have different names, or have evolved by professional communities that tend not to overlap.

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Some fields of mathematical modeling have used AI techniques for more than a decade, but the decade of the 1990s have seen explosive growth for at least two reasons: (1) the continued growth of both fields and their interdependencies, and (2) the inception of the World Wide Web and its meteoric rise into everyone's lives.

One of these areas is *Reasoning about Mathematical Models.* By its name, this is concerned with the application of AI to assist model formulation, management aids and analysis support [7]. Some results, however, apply the other direction. Advances in explaining consistency, redundancy and implied equalities in a system of inequalities apply to constraint logic programming (CLP). In fact, modeling paradigms have changed, shifting more towards the use of AI, while mathematical methods have become more sophisticated and more necessary to support the broader range of models couched in CLP.

Much of this overlaps with Symbolic Mathematical Computing (SMC)[4], particularly the qualitative analysis of model structures. The field of Pattern Recognition, originally regarded as a sub-discipline in AI, has shifted in the past decade to form its own body of knowledge using such mathematical foundations as category theory. Formulating patterns in problem solving [1] opens new approaches to reasoning about models and toward more automated formulation and reformulation, given problem information, which could be incomplete.

Automated theorem proving shows up in the works of Jeroslow [9], Williams [17], and the recent books by Chandru and Hooker [5] and McAloon and Tretkoff [10]. The theme is solving hard problems by representing a well formulated model in computational logic. The theoretical complexity is still there, but there has been great success in using this more natural statement of a problem. Some connections with optimization have been understood for decades, but new results have emerged in the past decade that deepen our understanding of problem formulation, re-formulation, pre-solving, analyzing, and finally solving hard problems. Partly, this is due to new computer technology, but it is more fundamental than that, and I see mathematical studies that will take computational complexity down new roads. One is the very difficult area of average-case complexity, which has been developing more rapidly in the past five years. Another road is that of burgeoning meta-heuristics [12], with great practical successes, with competing paradigms for unifying principles, notably *evolutionary computing* [2] and *tabu search* [6].

Problem solving in an uncertain world is reality. But, what do we mean by "uncertainty"? Classically, we have the debate between relative frequency versus Bayesian approaches to inference. For decades the AI community has also

used possibility theory, based on fuzzy sets, and the Dempster-Shafer Theory of Evidence. There are perhaps two dozen representations of uncertainty, each with some domain for which it seems to be the most realistic. (Some of these are reported by Hunter and Parsons [8], followed by a host of applications.) Modal logics and qualitative argument systems take a sharply different view of how to reason with uncertainty. The nature of the uncertainty and the goal of the reasoning are determining factors in what representation is most appropriate. Influence diagrams and belief nets [11, 13] explicitly integrate causality [14] with uncertainty. Perhaps subtly, we see that levels of modeling and the theorem proving reasoning must be viewed broadly across objects, relations and complex interactions.

The last decade was ushered in by Penrose's [15] provocative distinction between human and machine intelligence. His use of the Turing machine as the only theoretical model of computation was one of the fatal flaws in his argument. True neural networks, for example, are analogue computers that do not satisfy the Turing model. But, one can turn to the article by Blum, Smale and Shub [3] to see how Penrose's original argument might extend to continuous-time models of computation. Moreover, he answered other crucial criticisms in his sequel [16].

Not to be underestimated is the significance that the past decade has brought a quantum leap in the power of intelligent interfaces. The web has created major developments, particularly in user interfaces, and has created such urgent demands, that the level of research activity in mathematics and artificial intelligence must accelerate. Non-commercial benefits will accrue and change the way we see the human-computer interactions in a fundamental way. Viewing computers as having human reasoning qualities is the very heart of AI, and viewing humans as machines proved to be beneficial in many fields of science and medicine. Now a new view can emerge concerning the cultural elements that has the potential to render into fact what some writers have put into their fiction.

Mathematical reasoning is both the left and right side of this theory of problem solving. We should expect to see more new paradigms that foster new avenues for research in this far reaching and highly engaging area of mathematics and artificial intelligence.

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