Iterative Solvers for Coupled Fluid-Solid Scattering

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Abstract

The multigrid method is used for coupled fluid-solid scattering discretized by linear finite elements. Numerical results show that using Krylov methods as smoothers allows coarser spaces than with standard smoothers, such as Jacobi and Gauss-Seidel. Block diagonal preconditioning for the 2×2 block diagonal matrix of the coupled system is also considered. Both multigrid and block diagonal preconditioned iterations fail to converge for frequencies when the scatterer is at resonance. It is shown how to transform the system into an equivalent one to avoid the resonance and to recover the convergence of the iterations.

Key words: multigrid, preconditioning, resonance, Krylov, GMRES, smoothing

1 Introduction

The time-harmonic fluid-solid coupled problem gives rise to the Helmholtz equation in the fluid coupled with a Helmholtz-like equation in the solid. This paper is concerned with iterative methods for the systems of linear equations, obtained from a finite element discretization of the coupled system.

Scattering and propagation of waves is a classical area, which has engaged the interest of several generations of mathematicians, physicists, and engineers. Discretization estimates for time harmonic wave propagation are available, for

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example, for fluid in unbounded domains [3] using potential theory, for shell in fluid [6,7], using the inf-sup condition, and for 1D fluid-elastic scattering [15], using the Gårding inequality. An estimate in more than one dimension using the Gårding inequality, related to [15], was obtained in the thesis [21].

Efficient iterative solution of the systems of linear algebra is a challenge because of numerical loss of coercivity for high frequencies: there are many vectors, representing waves, which have zero or small residual away from the boundary, and the discrete system is indefinite. Early in multigrid, it was recognized that for indefinite problems, the coarsest grid needs to be fine enough [11,16,1]. For the Helmholtz equation, this means that the coarsest level needs to be fine enough to approximate the waves [19,10]. To relax this requirement, coarse basis functions defined by waves in a number of directions were proposed for multigrid methods [14,24], and, similarly, in the coarse problem in the FETI-H substructuring method [9]. For the same reasons, the use of a substructuring method was proposed to solve the coarse problem in multigrid [13]. An important observation was made in [8] that in multigrid, Krylov space methods as smoothers on coarse levels make it possible to use coarser coarse space than what would be otherwise required for standard smoothers, such as Gauss-Seidel. In this paper, we extend this observation to the case of the coupled problem.

Iterative methods for the time harmonic coupled problem have not been studied extensively. A substructuring method for the coupled problem was developed in [5] for the time dependent case. The FETI-H substructuring method was extended to the coupled time-harmonic equations in [17,18].

In this paper, we propose a multigrid method for the coupled problem, and a regularization technique to avoid resonance of the solid and consequent loss of convergence. The multigrid results are based on the thesis [21]. The regularization technique is a further development of an approach used in a substructuring algorithm [18]. It consists of a modification of the system matrix, and thus it is applicable to any iterative method for the coupled system.

The rest of this paper is organized as follows. In Section 2, we formulate the coupled problem. The variational formulation and the discretization are stated in Section 3. Section 4 contains a description of the multigrid method, and Section 5 presents the 2×2 block diagonal preconditioning. In Section 6, we present the technique to avoid the singular diagonal block. Finally, Section 7 contains computational results and their discussion.

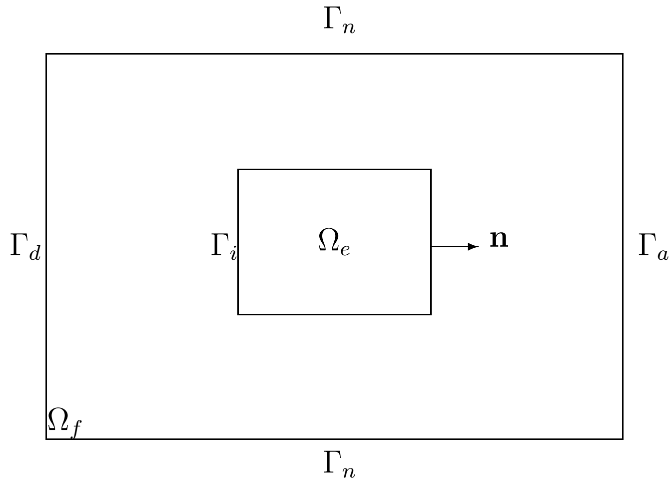


Fig. 1. Model coupled problem

2 The coupled problem

Consider the time harmonic acoustic scattering problem in a bounded domain $\Omega \in \mathbb{R}^n$, $n = 2$, or 3, filled with fluid with a speed of sound c_f and vibrating at angular frequency ω, cf. Figure 1. The acoustic field in the fluid is governed by the Helmholtz equation for the pressure p,

$$\Delta p + k^2 p = 0 \quad \text{in} \quad \Omega_f, \qquad k = \frac{\omega}{c_f}, \tag{1}$$

with the boundary conditions

$$\begin{aligned} p &= p_0 \quad \text{on} \quad \Gamma_d, \\ \frac{\partial p}{\partial \mathbf{n}} &= 0 \quad \text{on} \quad \Gamma_n, \\ \frac{\partial p}{\partial \mathbf{n}} + ikp &= 0 \quad \text{on} \quad \Gamma_a. \end{aligned} \tag{2}$$

Assuming an isotropic elastic medium, we have

$$\boldsymbol{\nabla} \cdot \boldsymbol{\tau} + \omega^2 \rho_e \mathbf{u} = 0 \quad \text{in} \quad \Omega_e, \tag{3}$$

where

$$\begin{aligned} \boldsymbol{\tau} &= \lambda I(\boldsymbol{\nabla} \cdot \mathbf{u}) + 2\mu \mathbf{e}(\mathbf{u}) && \text{is the stress tensor,} \\ e_{ij}(\mathbf{u}) &= \tfrac{1}{2}\left(\tfrac{\partial u_i}{\partial x_j} + \tfrac{\partial u_j}{\partial x_i}\right) && \text{is the strain tensor,} \end{aligned}$$

$\mathbf{u}$ is the displacement, ρ_e is the density, I is the $n \times n$ identity matrix, and λ and μ are the Lamé coefficients of the elastic medium.

The solid-fluid interface conditions are given by [25]

$$\left.\begin{aligned} \mathbf{n}\cdot\mathbf{u} &= \tfrac{1}{\rho_f\omega^2}\tfrac{\partial p}{\partial \mathbf{n}} \\ \mathbf{n}\cdot\boldsymbol{\tau}\cdot\mathbf{n} &= -p \\ \mathbf{n}\times\boldsymbol{\tau}\cdot\mathbf{n} &= 0 \end{aligned}\right\} \quad \text{on} \quad \Gamma_i. \tag{4}$$

The equations express, in turn, the continuity of displacement, the balance of the normal forces, and the zero tangential forces.

3 Variational formulation and discretization

The finite element discretization we use is standard [20]. A description is included for completeness. To derive a weak formulation of the acoustic elastodynamic model problem, define the spaces

$$V_f = \{\, p \,|\quad p \in H^1(\Omega_f), \quad p_\Gamma = 0 \text{ on } \Gamma_d \}\,, \tag{5}$$
$$V_e = \{\, \mathbf{u} \,|\quad \mathbf{u} \in (H^1(\Omega_e))^3 \}, \tag{6}$$

where the restrictions p_Γ denotes the trace. Multiplying equation (1) by a test function $\bar{q} \in V_f$, equation (3) by a test function $\bar{v} \in V_e$, and integrating by parts over Ω we obtain the following variational form of (1), (3), and (4): Find p, $(p - p_0, p_\Gamma) \in V_f$ and $\mathbf{u} \in V_e$, such that

$$-\int\limits_{\Omega_f} \boldsymbol{\nabla} p \boldsymbol{\nabla} \bar{q} + k^2 \int\limits_{\Omega_f} p\bar{q} - ik \int\limits_{\Gamma_a} p_\Gamma \bar{q}_\Gamma - \int\limits_{\Gamma} \rho_f \omega^2 (\mathbf{n}\cdot\mathbf{u}_\Gamma)\bar{q}_\Gamma = 0$$
$$-\int\limits_{\Omega_e} (\lambda(\boldsymbol{\nabla}\cdot\mathbf{u})(\boldsymbol{\nabla}\cdot\bar{\mathbf{v}}) + 2\mu\mathbf{e}(\mathbf{u}) : \mathbf{e}(\bar{\mathbf{v}})) + \omega^2 \int\limits_{\Omega_e} \rho_e \mathbf{u}\cdot\bar{\mathbf{v}} - \int\limits_{\Gamma} p_\Gamma(\mathbf{n}\cdot\bar{\mathbf{v}}_\Gamma) = 0$$
$$\forall\, \bar{q}, \bar{q}_\Gamma \in V_f,\ \bar{\mathbf{v}}, \mathbf{n}\cdot\bar{\mathbf{v}}_\Gamma \in V_e,$$

where p_0 on Γ_d is understood to be extended to a function in $H^1(\Omega_f)$.

Since λ and μ may be very large, use a scaling of the form $\mathbf{u} = s\mathbf{u}'$ and $\mathbf{v} = s\mathbf{v}'$, where s is a scalar. In computations we scale to

$$s = \frac{1}{c_f\sqrt{\rho_f \max\{\lambda, 2\mu\}}}. \tag{7}$$

Replacing V_f and V_e with conforming finite element spaces, we obtain the algebraic system

$$\begin{bmatrix} -\mathbf{S}_f + k^2\mathbf{M}_f - ik\mathbf{G}_f & -\rho_f\omega^2 s\mathbf{T} \\ -\rho_f\omega^2 s\mathbf{T}^t & -\rho_f\omega^2 s^2\mathbf{S}_e + \rho_e\rho_f\omega^4 s^2\mathbf{M}_e \end{bmatrix} \begin{bmatrix} \tilde{p} \\ \tilde{u} \end{bmatrix} = R \qquad (8)$$

In (8), $\tilde{p}$ and $\tilde{u}$ are the algebraic representations of p and u, i.e., p and u are the finite element interpolations of $\tilde{p}$ and $\tilde{u}$, respectively. The matrix blocks in (8) are defined by [21]

$$\mathbf{S}_f = \int_{\Omega_f} \boldsymbol{\nabla} p_h \boldsymbol{\nabla} q_h, \qquad \mathbf{M}_f = \int_{\Omega_f} p_h q_h, \qquad \mathbf{G}_f = \int_{\Gamma_a} p_h q_h,$$

$$\mathbf{S}_e = \int_{\Omega_e} \lambda(\boldsymbol{\nabla}\cdot\mathbf{u}_h)(\boldsymbol{\nabla}\cdot\mathbf{v}_h) + 2\mu\mathbf{e}(\mathbf{u}_h) : \mathbf{e}(\mathbf{v}_h),$$

$$\mathbf{M}_e = \int_{\Omega_e} \mathbf{u}_h \cdot \mathbf{v}_h, \qquad \mathbf{T} = \int_{\Gamma} p_h(\mathbf{n}\cdot\mathbf{v}_h).$$

To obtain error estimates we choose the scalar s so that the leading terms of equation (8) differ by a factor of k^2

$$s = \frac{1}{c_f k\sqrt{\rho_f \max\{\lambda, 2\mu\}}}.$$

4 Multigrid for the coupled problem

In this section, we present the multigrid algorithm to solve the coupled problem (8). The algorithm follows standard multigrid techniques, see, e.g. [4].

We have obtained the discrete system (8) by bilinear elements on a uniform mesh. We write (8) as $Au = f$, where the coefficient matrix A is complex symmetric, i.e., not Hermitian. For large values of the wave number k, the system of equations becomes highly indefinite. Coarsening was chosen to be a factor of 2 and prolongations defined by the natural embedding of the finite element spaces. The restrictions are defined variationally, i.e., as the transposes of the prolongations. All nodes have 3 degrees of freedom, namely the pressure p, the displacement in the x direction, u_x, and the displacement in the y direction, u_y. A degree of freedom at a node is active, if it has nonzero rows or column entries in the system matrix. For the fluid nodes, only p is active. For the solid nodes, the displacements u_x and u_y are active. Nodes on the wet interface have all three degrees of freedom active. The prolongation is applied separately to each of the three fields. For the nodes inside the fluid or inside the solid, the prolongation in each active field is defined by the bilinear interpolation stencil

$$\begin{bmatrix} \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ \frac{1}{2} & 1 & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \end{bmatrix}$$

The elastic obstacle is assumed aligned with the grid lines of the coarsest grid. For a coarse node at a straight part of the wet interface, we use the stencil

$$\begin{bmatrix} \frac{1}{4} & \frac{1}{2} & 0 \\ \frac{1}{2} & 1 & 0 \\ \frac{1}{4} & \frac{1}{2} & 0 \end{bmatrix},$$

to average neighboring values. For a coarse node which is a corner of the wet interface, we use stencils like

$$\begin{bmatrix} \frac{1}{4} & \frac{1}{2} & 0 \\ \frac{1}{2} & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Finally, for inside corner of the wet interface, we use the stencil

$$\begin{bmatrix} \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ \frac{1}{2} & 1 & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{2} & 0 \end{bmatrix}.$$

Given the finest level matrix A_0 from finite elements, the coarse matrices are created variationally by

$$A_{l+1} = (P_{l+1}^l)^T A_l P_{l+1}^l,$$

where P_{l+1}^l is the prolongation from the coarser level $l+1$ to the finer level l.

One iteration of standard *V-cycle* multigrid algorithm $x \leftarrow MG(x, b)$, solving $Ax = b$, can be described in abstract terms as follows. Set $A_0 = A$ and $MG = MG_0$, where for $l = 1, ..., L-1$, MG_l is defined by

- Pre-smoothing: Perform m_1 smoothing steps on $A_l u_l = f_l$
- Coarse grid correction
  - · compute residual $r_l = f_l - A_l u_l$
  - · restrict $f_l = (P_{l+1}^l)^T r_l$

  · if $l+1 = L$ then solve coarse grid equation by a direct method, else perform m_c iterations of $u_{l+1} \leftarrow MG_{l+1}(0, f_{l+1})$
  · interpolate $v_l = P_l v_{l+1}$
  · correct the solution on level l by $u_l \leftarrow u_l + v_l$
- Post-smoothing: Perform m_2 smoothing steps on $A_l u_l = f_l$

Here, m_1 and m_2 are the number of pre smoothing and post smoothing steps per multigrid level, and m_c is the multigrid cycle parameter. As a smoother we use Generalized Minimum Residual method (GMRES) Krylov method. GMRES is a projection method that approximates the solution of a system of linear equations $Au = f$ for u, by minimizing the residual norm $\|f - Au\|$ over the m-th Krylov subspace $\{x_0 + \text{span}\{r, Ar, ..., A^{m-1}r\}\}$ [2,22,23]. The method requires storing all search directions and the solution of an $(m+1) \times m$ least squares problem in iteration m [22]. We will also investigate smoothing by BICGSTAB [22], which uses a short recursion instead of storing all search directions to save computational work, but does not guarantee a monotone decrease of the residual norm. Finally, we also consider smoothing by GMRES preconditioned by the inverse of the lower triangular part of the matrix.

5 Iterative solution by 2×2 block diagonal preconditioning

We also consider solving the coupled system by GMRES preconditioned by ILU applied to the diagonal blocks of (8). Let $L_f R_f$ be approximate factorization of the matrix $-\mathbf{S}_f + k^2\mathbf{M}_f - ik\mathbf{G}_f$, which is the diagonal submatrix for the fluid part, and $L_e R_e$ be approximate factorization of the diagonal submatrix for the solid part, $-\rho_f \omega^2 s^2 \mathbf{S}_e + \rho_e \rho_f \omega^4 s^2 \mathbf{M}_e$. We then use the block diagonal matrix

$$\begin{bmatrix} [L_f R_f]^{-1} & 0 \\ 0 & [L_e R_e]^{-1} \end{bmatrix}$$

as a right preconditioner to GMRES.

6 Regularization by a radiation-like condition on the wet interface

The matrix $-\mathbf{S}_e + \omega^2 \mathbf{M}_e$ in the coupled system (8) is singular for some frequencies ω. This causes iterative methods for the coupled system to fail at or near those frequencies. One would clearly expect trouble for methods that rely on the diagonal block being nonsingular, such as block diagonal preconditioning. However, computational examples in Section 7 show deterioration of convergence up to failure to converge also for GMRES with multigrid preconditioning, which does not make any nonsingularity assumption. Therefore, it is important to remove the singularity of the diagonal block in any case.

To prevent the second diagonal block of system matrix from becoming singular in the case of resonance, we replace the system (8) by an equivalent system by adding to the second block of equations a linear combination of the equations in the first block, so the system matrix in (8) becomes

$$\begin{bmatrix} -\mathbf{S}_f+k^2\mathbf{M}_f-ik\mathbf{G}_f & -\rho_f\omega^2 s\mathbf{T} \\ -\rho_f\omega^2 s\mathbf{T}^t-i\beta\mathbf{T}^t(-\mathbf{S}_f+k^2\mathbf{M}_f-ik\mathbf{G}_f) & -\rho_f\omega^2 s^2\mathbf{S}_e+\rho_e\rho_f\omega^4 s^2\mathbf{M}_e+i\beta\rho_f\omega^2\mathbf{T}^t\mathbf{T} \end{bmatrix}.$$

This has the effect of adding to the second diagonal block an imaginary multiple of a positive definite matrix, which shifts the possible offending zero eigenvalue of the diagonal block into the complex plane. The added term is similar to the artificial radiation condition (2). The constant β is chosen as

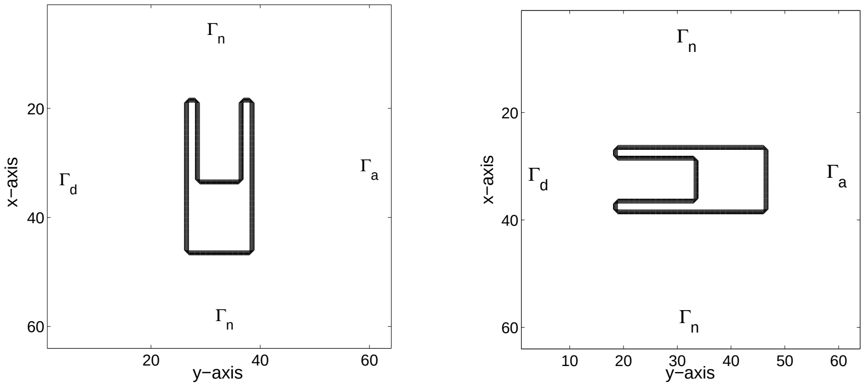
$$\beta = \beta_0 \frac{\omega\sqrt{\rho_e(\lambda+2\mu)}}{\|\mathbf{T}\|_1},$$

so that the equation has consistent physical units and the added term has similar strength as the term arizing from the radiation condition (2). We will experiment with the choice of the dimensionless parameter β_0.

7 Computational results

We consider a 2D model problem as in Fig. 1. The domain is the square $(0,1)\times(0,1)$. The obstacle in the channel is set up in the center of the fluid domain as a square of size $0.2m$, unless specified otherwise in some problems. In all computational examples, the size of the domain in the x and y-direction is chosen to be always 1 m. The boundary condition on Γ_d is $p_0(x,y) = 1$. The origin of the coordinate system is assumed to be in the lower left corner of Ω. The fluid medium is water with density $\rho_f = 1000 kg\, m^{-3}$ and speed of sound $c_f = 1500 m\, s^{-1}$. The elastic medium is aluminum with density $\rho_e = 2700 kg\, m^{-3}$ and Lamé elasticity coefficients are $\lambda = 5.5263 \cdot 10^{10} N\, m^{-2}$, and $\mu = 2.595 \cdot 10^{10} N\, m^{-2}$. The fluid domain and the solid domain are discretized by standard bilinear square finite elements $Q1$ on a uniform mesh with mesh size h. The discrete system is scaled according to (7). The iterations were terminated when the relative residual reached 10^{-6}. Then the average residual reduction per smoothing step was computed as

$$\text{residual reduction} = \left(\frac{\text{residual last}}{\text{residual initial}}\right)^{1/\text{number of smoothing steps}}$$



(a) (b)

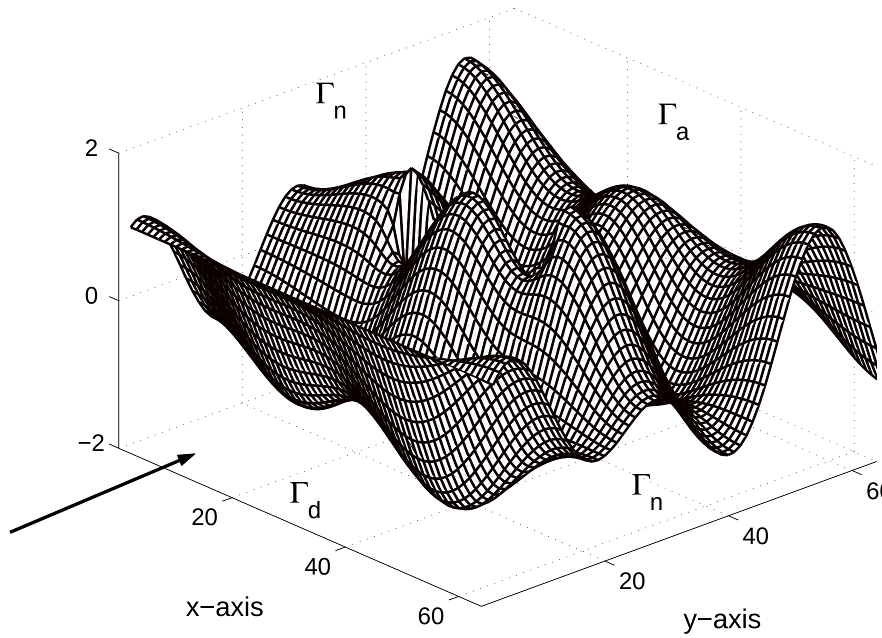
Fig. 2. (a) Contour of obstacle of size 0.4 m in x and 0.2 m in y direction. Gap on y axis of 0.5 or 50% and 0.4 or 40% in x and y direction, respectively. (b) Contour of obstacle of size 0.2 m in x and 0.4 m in y direction. Gap on x axis of 0.4 or 40% in x and 0.5 or 50% in y direction, respectively.

We present results and convergence factors, where the number of pre smoothing steps always equals the number of post smoothing steps. The problem on the coarsest level was solved directly. We also use GMRES preconditioned by one full multigrid cycle.

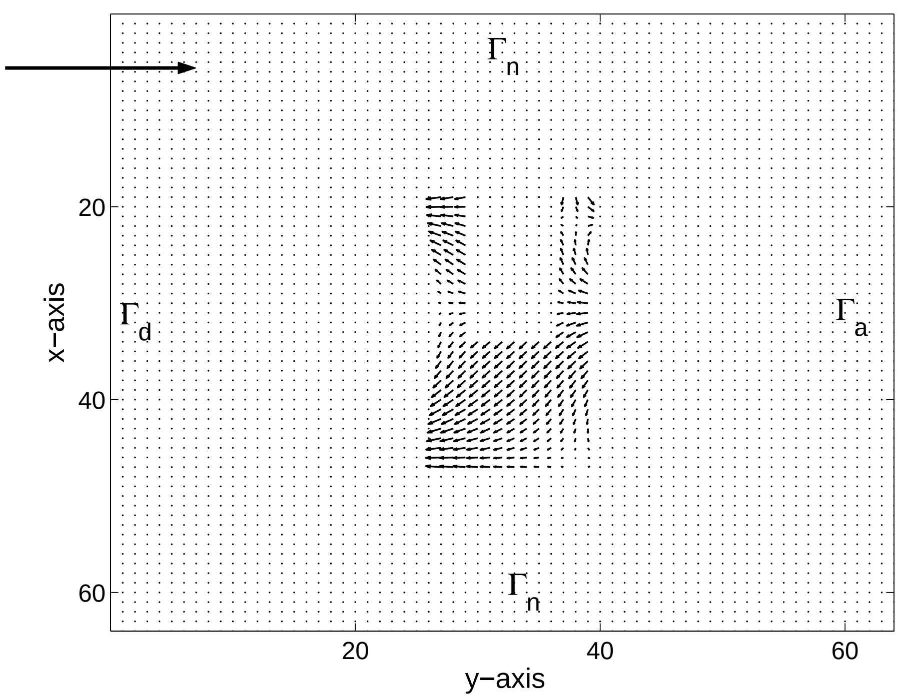
Tests are run for different mesh sizes h. When the mesh is refined the frequency is increased, so that the value of k^3h^2 is kept constant. For the Helmholtz equation, it is known that the faster decrease of h compared to the increase in k is needed to avoid pollution by the phase error and to keep the error decreasing with h, cf., [12].

We display the solution for some cases considered. We display the real part of amplitude for pressure as elevation above the xy plane, and the displacement in x and y direction as equally spaced arrows based at the original configuration. For many applications the computational domain is not always symmetric. As a model problem, we consider a rectangular scatterer that has a rectangular indentation, which we call "u-shaped", cf. Figure 2. Figures 3 and 4 show the real part of the solution for a u-shaped obstacle, with the gap in the y and the x direction, respectively.

In Figure 5, we compare the multigrid performance for several numbers of coarse meshes, several meshsizes, and four different smoothers. We observe that multigrid with Gauss-Seidel as smoother diverges when there are too coarse meshes, and that smoothing by GMRES results in faster convergence than smoothing by BICG-STAB. However, smoothing by GMRES is more expensive than BICG-STAB, which is in turn more expensive than Gauss-Seidel. The divergence of multigrid with Gauss-Seidel smoothing can be explained by the fact that Gauss-Seidel iteration amplifies the smooth error components,

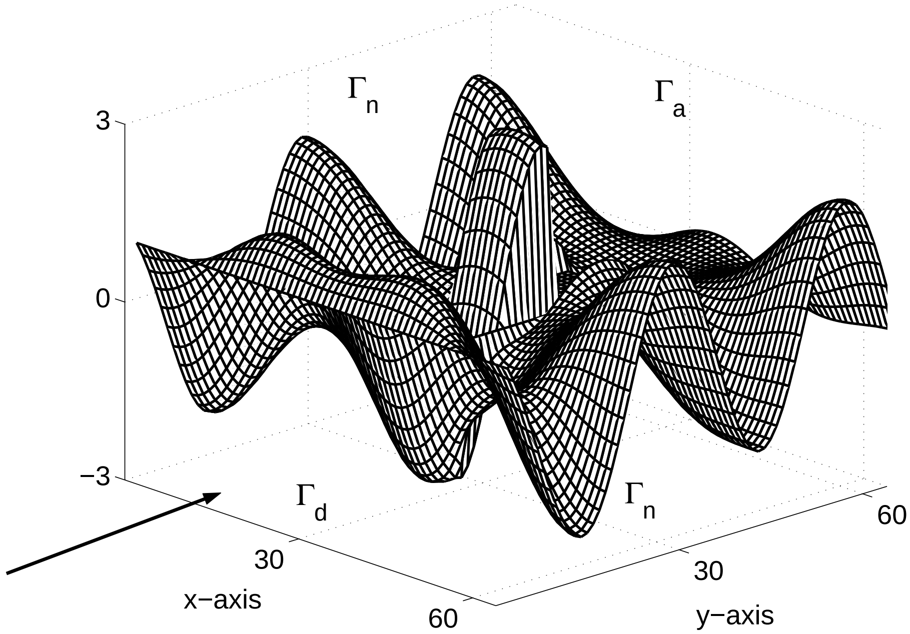


Fluid pressure

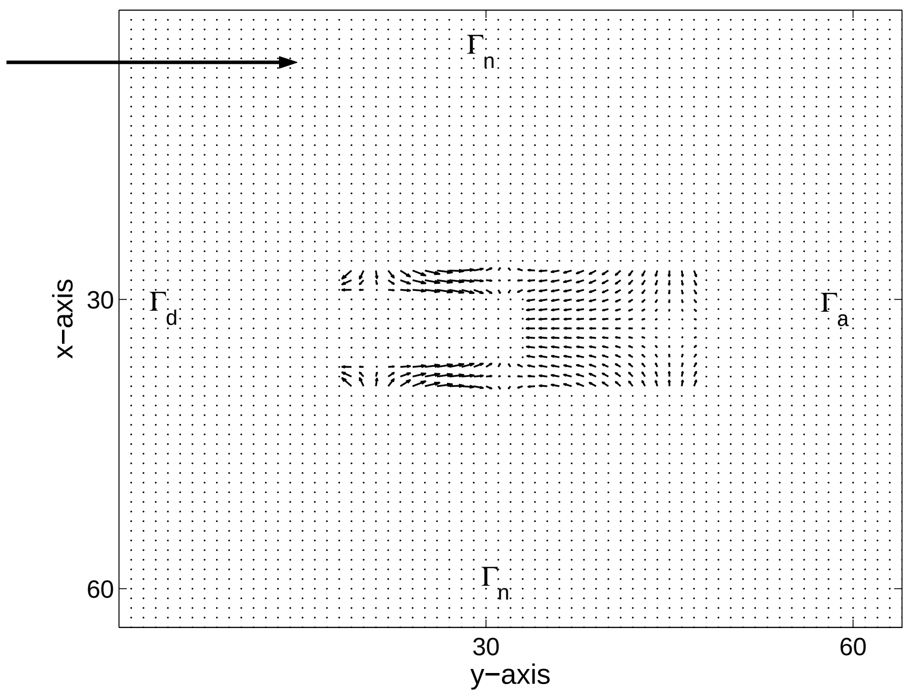


Solid displacement

Fig. 3. Solution for a 64×64 mesh u-shaped obstacle 0.2 in x and 0.4 in y direction. Obstacle has a gap on y axis of 0.4 and 0.5 in the x and y direction respectively, and $k = 15$.



Fluid pressure



Solid displacement

Fig. 4. Solution for a 64 × 64 mesh ushaped obstacle 0.2 in x and 0.4 in y direction with a gap on the x axis of 0.4 and 0.5 in the x and y direction respectively, $k = 15$

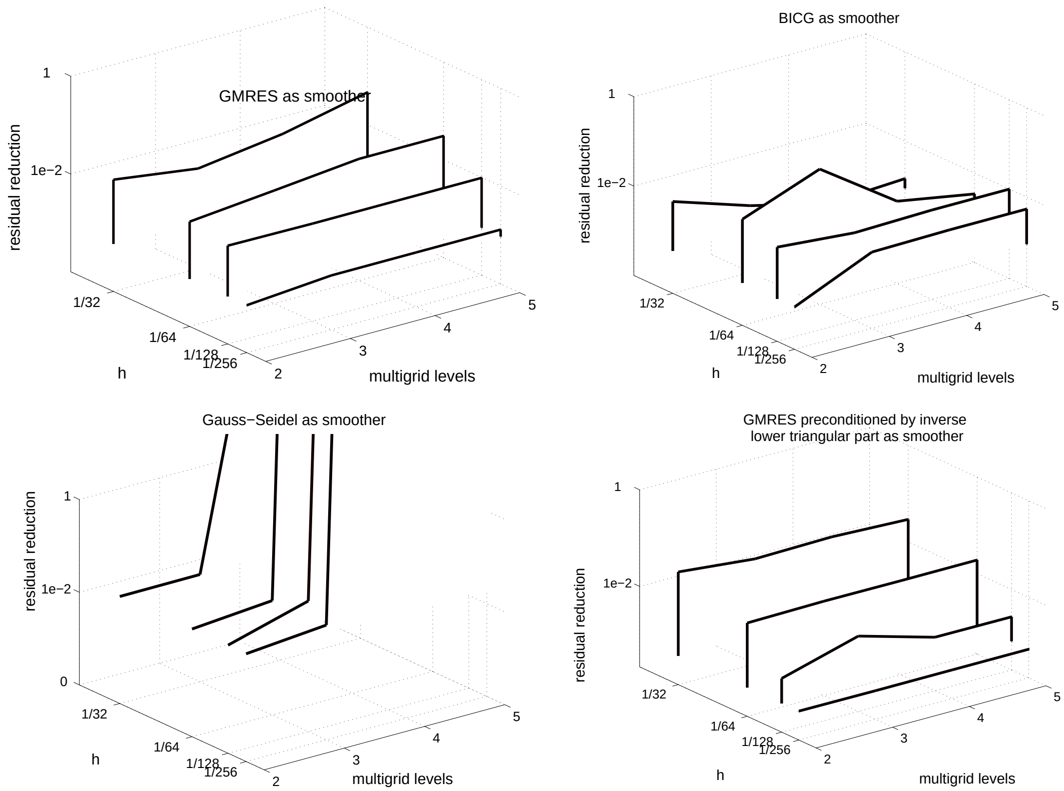


Fig. 5. Multigrid performace for decreasing h, k^3h^2 constant, adding coarse meshes, square obstacle in the middle of waveguide of size 0.2 m by 0.2 m, 10 smoothing steps

similarly as in [8]. Multigrid with smoothing by GMRES preconditioned by inverse of lower triangular part of A is found to be the most robust one and give best results, however it requires the most memory.

Figure 6 contains a comparison of multigrid convergence for several smoothers, increasing the number of smoothing steps, and decreasing h with k^3h^2 constant. We observe that as the mesh size decreases, the convergence of the iterations improves. This is due to the fact that k increases slower than $1/h$, so the problem is not as much indefinite for small h. Increasing the number of smoothing steps per multigrid level improves convergence except for Gauss-Seidel, as expected.

Figure 7 (a) contains a comparison of multigrid convergence for an obstacle 0.2 in the middle with a gap on y axis of 0.4 and 0.5 in the x and y direction respectively. We increase the number of smoothing steps and keep the number multigrid levels constant at 2, and decrease h with k^3h^2 constant. We observe that as the mesh size decreases, the convergence of the iterations improves. The results are comparable to the case of the obstacle without a gap. However, we observe that more smoothing steps are required to obtain convergence than for the case without a gap. In figure 7 (b) we look at multigrid performance for

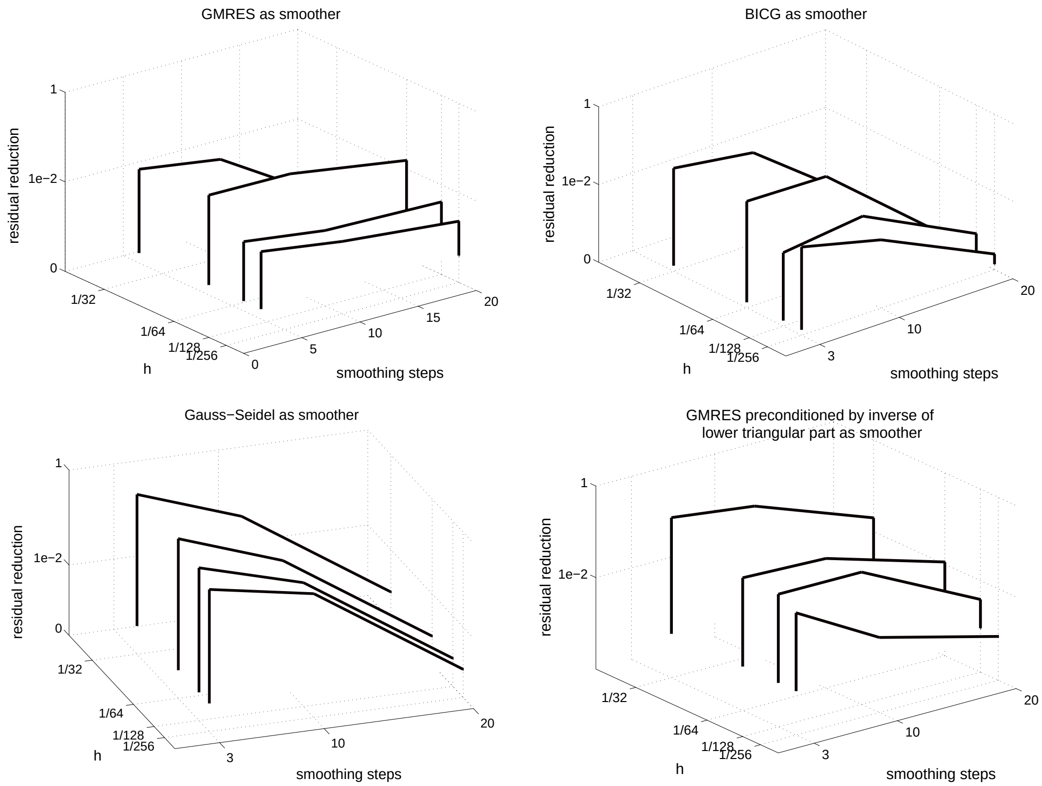
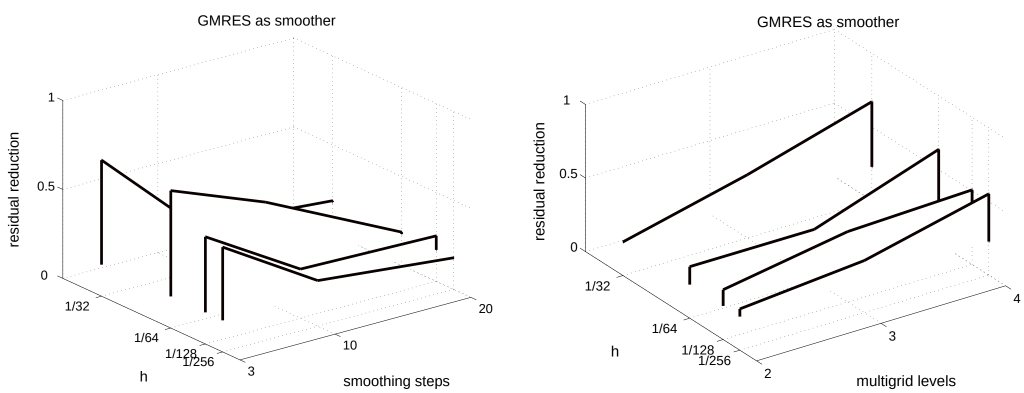


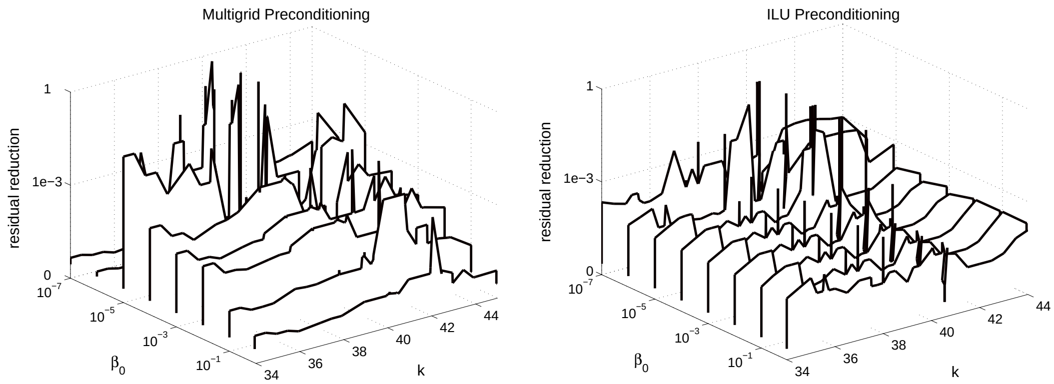
Fig. 6. Multigrid performance for decreasing h, k^3h^2 constant, and increasing smoothing steps, 2 multigrid levels, square obstacle of size 0.2 m by 0.2 m



(a) (b)

Fig. 7. Multigrid performance for decreasing h, k^3h^2 constant, obstacle size 0.2 with a gap on y axis of 0.4 and 0.5 in the x and y direction respectively. (a) Increase number or smoothing steps while keeping the multigrid levels at 2. (b) Increase number of multigrid levels and keeping the number of smoothing steps at 20.

several numbers of coarse meshes, several meshsizes, and GMRES as smoother. We observe that for the onbstacle with a gap increasing the number of coarse meshes convergence is much slower than in the case of an obstacle without a gap.



(a) (b)

Fig. 8. Residual reduction by 3 iterations, square scatterer 0.2×0.2, mesh 200×200, varying frequency around the first eigenvalue of the scatterer, and varying the regularization parameter β_0. (a) GMRES with multigrid preconditioning, 20 smoothing steps and GMRES smoother (b) GMRES with ILU preconditioning of the two diagonal blocks

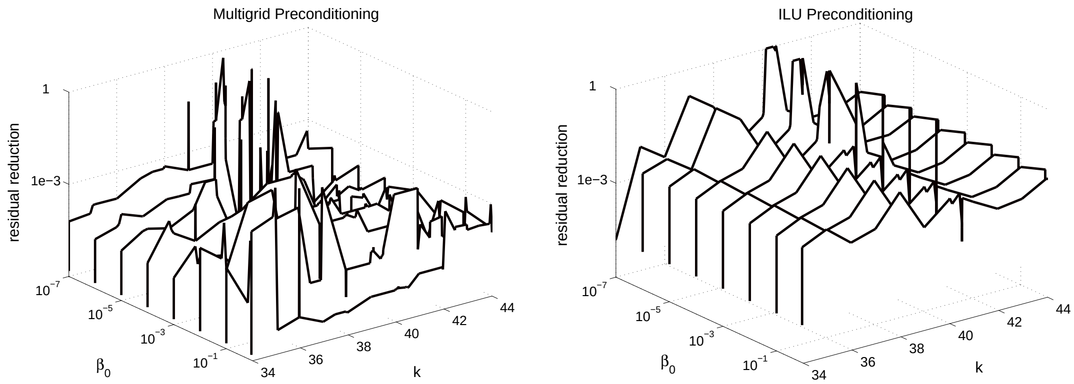
In Figure 8, we look at the case when k is equal or is close to a value that makes the elastic matrix singular. The scatterrer is 0.2 m by 0.2 m in the center of the wave guide. We have tested two preconditioners: one 2 level multigrid cycle with 2 iterations and 20 smoothing steps, and Incomplete LU decomposition (ILU), applied in each of the two diagonal blocks separately. We have used the routine ILU from Matlab with drop tolerance.

We see that in both cases, for small β_0 we obtain a residual reduction close to 1 and the iterations do not converge. The convergence is better for the multigrid preconditioner; possibly because it couples the fluid and the elastic block, so it does not suffer from the singularity of the elastic block as much.

In Figure 9, we consider BICG with ILU preconditioning of the two diagonal blocks. the scatterrer is 0.2 m by 0.2 m in the center of the wave guide. We see that the results are comparable to GMRES smoothing.

8 Conclusion

We present numerical results for a mutligrid method applied to the discrete coupled fluid-solid scattering problem. The results show that Krylov smoothers, in particular GMRES, make robust algorithms. We compare GMRES as smoother with BICG as smoother and find that the results are comparable. We also investigate GMRES preconditioning with multigrid preconditioning. We observe that we obtain better convergence for multigrid preconditioning possibly because it couples the fluid and the elastic block, and it does not suffer from



(a) (b)

Fig. 9. Residual reduction by 3 iterations, square scatterer 0.2×0.2, mesh 200×200, varying frequency around the first eigenvalue of the scatterer, and varying the regularization parameter β_0. (a) BICG with multigrid preconditioning, 20 smoothing steps and BICG smoother (b) BICG with ILU preconditioning of the two diagonal blocks

the singularity of the elastic block as much. For the case of a square obstacle and the case of an obstacle with a gap, we investigate convergence in two cases. First we vary the number of smoothing steps and keep the number of multigrid levels constant, and second by increasing the number of multigrid levels and keeping the number of smoothing steps constant. For both types of obstacles we observe that more smoothing steps and less multigrid levels give best convergence rates.

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